Report on the Hausdorff Trimester Program

Geometry and Dynamics of Teichmüller Spaces

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Topics

1. Geometric properties of Teichmüller space
2. Flat surfaces and billiards
3. Interval exchange transformations
4. Veech surfaces and Veech groups
5. Origamis, also of infinite type
6. Invariant measures for the Teichmüller flow
7. Geometric properties of mapping class groups

Goals

The goal was to bring together experts on various aspects of Teichmüller theory including algebraic geometry, low-dimensional topology, geometric group theory and dynamical systems and foster the interaction of these groups. Moreover, young researchers, notably graduate students and recent graduates, should fully participate in all activities, have the opportunity to present their results, learn techniques beyond their immediate experience and initiate new research projects.

Organization

The activity began with a workshop on "Subgroups of mapping class groups in the first week in Mai. The goal of the workshop was to report on recent progress about geometric and topological properties of subgroups of the mapping class group of a surface of finite type or a three-manifold. Examples of such groups are the Torelli group or the handlebody group. Speakers included Ken Bromberg (Utah), Richard Canary (University of Michigan), Dan Margalit (Georgia Tech), Kasra Rafi (University of Oklahoma), Chris Leininger (University of Illinois) as well as a number of postdocs (mainly from France) and finishing graduate students. There was a lively exchange of ideas during the workshop.

During the month of Mai and June, there were many scientific activities on all levels. The activities were organized partially official, in form of traditional seminar talks, partially via a wiki for the activity. The wiki was used to announce formal and informal talks, as a platform for discussion and to request lectures on specific topics. Moreover, it was used by the participants to organize small working groups to go through an article or preprint and to discuss the results and methods, with a view towards future research.

I think that this form of organizing the activities of the program worked very well. It integrated the graduate students (there was a good number of gradutate students participating) and created a stimulating friendly working atmosphere where everyone was invited to talk about his research, to present work of other people he or she found interesting and to engage in discussions.

The highlight of the activity was the main workshop in June. The speakers were chosen to cover a large spectrum of topices related to the program, ranging from algebraic geometry to dynamical systems. They were on all levels of their career. Senior speakers included Curtis McMullen, Howard Masur, Pascal Hubert, Giovanni Forni and Alex Eskin (violating the rule that an organizer should not speak due to a spectacular result he had just proven). Among the junior speakers were some graduate students (Thomas Koberda, Ronen Mukamel) as well as very recent PhDs (John Chaika, Pat Hooper, Carlos Matheus). Between the talks there was a generous amount of time for discussions.

The feedback obtained from the participants of the workshop was overwhelming. Everyone thought that as a scientific event, it was very intense, very inspiring and very enjoyable.

Results

1. Ratner's theorem for the $SL(2,(R))$-action on moduli space
2. Progress towards understanding pseudo-Anosov mapping classes with small dilatation
3. Constructing hyperbolic spaces in Teichmüller space
4. Estimates of Lyaponov exponents for the Teichmüller flow
5. Fine fellow-travel properties for Teichmüller geodesics with nearby endpoints
6. Classification of the locus with degenerate Lyaponov spectrum for small genus